INTERFERENCE IN F-M RECEIVERS

Review of equations covering interference-suppressing ability when desired and undesired signals have the same average frequency, with experimental verification by measurements on a representative commercial receiver

By ROBERT N. JOHNSON
Collingswood, N. J.

THERE has been developed a theory by Reich[1], Pollack[2], and several others concerning the ratio of interference to desired signal in the audio output of an idealized frequency-modulation receiver for known ratios of interfering to desired input signals. The object of this investigation was to determine whether or not an average commercial frequency-modulation receiver adheres to this theory regarding interference.

The type of interference which was studied and applied in measuring the suppression ability of the receiver under test was that which results from two signals having the same average frequency, one being considered as the desired signal and the other as an interfering signal. The conditions chosen represent an interference problem at its worst.

Theory of Single-Channel Interference

For convenience, consider the desired signal to be of unit amplitude, and the interfering signal to be of amplitude a. It is assumed that if a were comparable to "unity," the interference would be so great that no one would attempt to listen to the desired signal. Consequently, this case has little practical interest. However, when the desired signal is at a modulation lull, the interfering signal, even though relatively small in magnitude, produces an undesirable psychological effect, since it can now be heard together with the desired audio output. It is under these general conditions that the interference suppression of a commercial receiver was studied.

A term which occurs quite frequently in the analysis of single-channel interference is instantaneous frequency. Assuming a voltage of the form $e = A \cos \phi$, a customary and suitable definition of instantaneous frequency is $\omega = d\phi/dt$, where ω is the instantaneous angular frequency and $d\phi/dt$ represents the rate of change of the angular displacement with respect to time.[3] It is this definition that will be used.

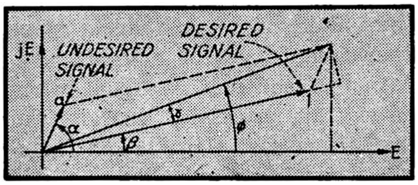

FIG. 1—Vector diagram for desired and undesired f-m signals, both having the same frequency

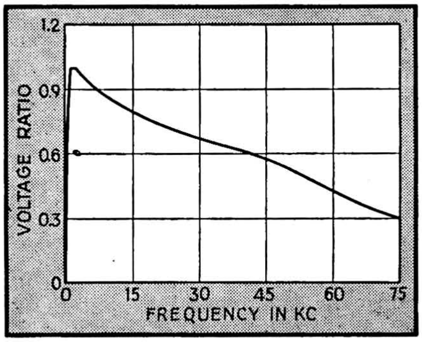

FIG. 2—Frequency response of output system into which audio system of f-m receiver was fed for visual indication of interference wave form

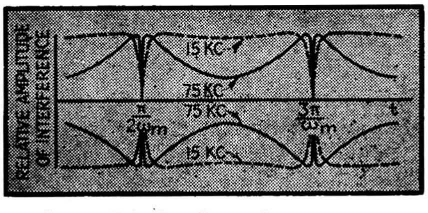

FIG. 3—Calculated interference pattern envelopes for deviations of 15 and 75 kc

The desired signal is not modulated, the condition of a modulation lull being assumed; only the carrier is present. The undesired signal is frequency-modulated with a frequency deviation of $\pm\overline{\Delta\omega}$ and at a modulation frequency of ω_m which is much less than the carrier frequency.

Referring to Fig. 1, the desired and undesired signals at some instant of time are respectively 1 cos β and a cos α. The following expressions may be written

$$\frac{d\beta}{dt} = \omega_0 = \text{constant carrier frequency} \quad (1)$$

$$\beta = \int \omega_0 \, dt = \omega_0 t + \theta \quad (2)$$

$$\frac{d\alpha}{dt} = \omega'(t) = \omega_\theta + \overline{\Delta\omega} \cos(\omega_m t) \quad (3)$$

which represents the frequency variations of a frequency-modulated signal. Then

$$\alpha = \int \omega'(t) dt = \left[\omega_0 t + \frac{\overline{\Delta\omega}}{\omega_m} \sin(\omega_m t + \theta) \right] \quad (4)$$

The output of a frequency-modulation receiver should be proportional to the instantaneous frequency of the input signal. In the present case, the instantaneous frequency resulting from the superposition of two input signals is to be determined. Referring to Fig. 1 again, the instantaneous frequency of the resultant signal is seen to be $\omega = d\phi/dt$. The problem, therefore, is to solve for the value of ω.

It can be seen by inspection that $\phi = \beta + \gamma$. Now by simple trigonometric relationships we can write

$$\gamma = \tan^{-1} \frac{a \sin (\alpha - \beta)}{1 + a \cos (\alpha - \beta)} \qquad (5)$$

or, since a is much less than unity

$$\gamma \cong a \sin (\alpha - \beta) = a \sin \left[\frac{\overline{\Delta\omega}}{\omega_m} \sin (\omega_m t) \right. \qquad (6$$

Then

$$\phi = \beta + a \sin \left[\frac{\overline{\Delta\omega}}{\omega_m} \sin (\omega_m t) \right] \qquad (7)$$

By definition

$$\omega = \frac{d\phi}{dt} = \omega_0 +$$

$$[a\, \overline{\Delta\omega} \cos (\omega_m t)] \cos \left[\frac{\overline{\Delta\omega}}{\omega_m} \sin (\omega_m t) \right] \qquad (8)$$

where

ω = resultant instantaneous angular frequency of input signals
a = magnitude of undesired signal, equal to the ratio of interfering to desired input signals since the desired signal is of unit amplitude
$\overline{\Delta\omega}$ = maximum angular frequency deviation of interfering signal
ω_m = angular frequency of modulation of interfering signal
t = time in seconds

Analysis of Discriminator Output

The resultant instantaneous angular frequency as given by Eq. (8) is seen to be composed of both a constant and a varying term. In a frequency-modulation receiver, the output of the discriminator is proportional to the variations in the instantaneous angular frequency. Consequently, the varying term of Eq. (8) is important in this analysis. It is equivalent to a sinusoidal variation with magnitude $[a\, \overline{\Delta\omega} \cos (\omega_m t)]$ and frequency $d/dt\, [(\overline{\Delta\omega}/\omega_m) \sin (\omega_m t)] = \overline{\Delta\omega} \cos (\omega_m t)$, with both the magnitude and the frequency varying sinusoidally with time. This varying frequency will be defined as the interference frequency and will be denoted by the symbol f_{int}. Thus, $f_{\text{int}} = [\Delta\omega \cos (\omega_m t)]/2\pi$.

Considering Eq. (8) once again, the resultant instantaneous angular frequency varies sinusoidally in magnitude about the mean carrier frequency ω_0. These variations are from ω_0 to $\omega_0 + a\, \overline{\Delta\omega}$ and back to ω_0 for half a period of the modulating wave. At any particular instant of time, deviation of the resultant instantaneous angular frequency is occurring at a definite frequency which is the interference frequency. Thus we have a frequency (the resultant instantaneous angular frequency) varying sinusoidally in magnitude at some other frequency (the angular modulation frequency). Also, when viewed at successive instants of time, these variations are in themselves varying sinusoidally at still another frequency, the interference frequency.

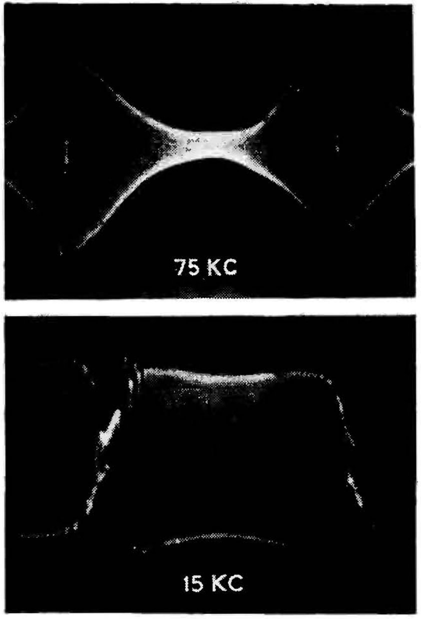

FIG. 4—Oscilloscope patterns obtained under same conditions as represented by Fig. 3

Allowances for Deemphasis

The results thus far have given us, theoretically, the wave form of the interference in the output of a frequency-modulation receiver, since in this type of receiver the audio output is proportional to the variations in instantaneous frequency as given in Eq. (8). However, in practice the frequency response of the receiving system is not flat. This is due to the frequency limitations of the audio system and to the standard deemphasis circuit placed at the output of the discriminator to restore the modulation to what it was before standard RMA preemphasis at the transmitter. This preemphasis of the higher-frequency components is proportional to $\sqrt{1 + (\tau\omega_1)^2}$, where τ is the standard 100-microsecond time constant and ω_1 is the angular frequency. The reciprocal of this proportionality factor is effected in the receiver by the deemphasis circuit. Consequently, the interference in the audio output is actually proportional to

$$\frac{[a\, \overline{\Delta\omega} \cos (\omega_m t)]}{\sqrt{1 + (\tau\omega_1)^2}} \cos \left[\frac{\overline{\Delta\omega}}{\omega_m} \sin (\omega_m t) \right] = \frac{a\, 2\pi f_{\text{int}}}{\sqrt{1 + (\tau\, 2\pi f_{\text{int}})^2}} \cos \left[\frac{\overline{\Delta\omega}}{\omega_m} \sin (\omega_m t) \right] \qquad (9)$$

The ratio of interference to desired signal in the output of the receiver as considered in this analysis is N/S, where N is peak amplitude of interference in audio output of f-m receiver due to superposition of a small, undesired, frequency-modulated signal on an unmodulated desired signal, both signals being of the same average frequency, and S is peak amplitude of the desired signal alone, frequency-modulated, in audio output of f-m receiver.

The problem is to determine N/S for various ratios of input signals. The ratio of input signals is given by a since the desired signal is considered to be of unit amplitude. In this analysis, the standard maximum frequency deviation of ±75 kc is assumed to correspond to the full permissible range of a modulating signal at the input of the system. It follows that the fractional utilization of this full range by the interfering signal in the audio output is

$$\frac{a\, 2\pi f_{\text{int}}}{2\pi \times 75{,}000 \sqrt{1 + (\tau\, 2\pi f_{\text{int}})^2}}$$

Thus the ratio of interference to desired signal in the output may be written as

$$\frac{N}{S} = \frac{a f_{\text{int}}}{75{,}000 \sqrt{1 + (\tau\, 2\pi f_{\text{int}})^2}} \qquad (10)$$

or, if $(\tau\, 2\pi f_{\text{int}})^2$ is much larger than unity, the above expression becomes

$$\frac{N}{S} = \frac{a}{2\pi \times 75{,}000 \tau} = \frac{a}{\tau\, \Delta\omega} \qquad (11)$$

For $\tau = 100\ \mu$sec, this reduces to $N/S = a/47$.

Practical Considerations

For input signal sources two signal generators were employed, one of which could be frequency-modulated. These were connected to the input terminals of the commercial frequency-modulation receiver under test. This particular receiver included a deemphasis circuit but not an audio amplifier. The audio output of the receiver was fed into the output system, which consisted of the following units given in the order of connection to the receiver: (1) a calibrated voltage-dividing network; (2) a high-gain voltage amplifier; (3) a high-pass filter circuit; (4) an oscilloscope.

The voltage-dividing network was used to produce an equal deflection on the oscilloscope screen for the cases of interference and desired signal respectively in the audio output of the receiver. It was thus possible to determine N/S for various prescribed input conditions. Because of the small magnitude of the interference voltage it was necessary to provide the high-gain voltage amplifier. Also, because of the small magnitude of the interference voltage, hum pickup of supply power frequencies and harmonics thereof became objectionable. The high-pass filter was inserted to eliminate this trouble. The oscilloscope was used to give a visual indication of the interference wave form and also as an indicator of its peak amplitude.

The over-all frequency response of the output system is given in Fig. 2. This includes the amplifying circuits of the oscilloscope. The effect of the filter circuit in cutting off the lower frequencies is evident. The linear frequency scale was used to give direct comparison with the cathode-ray oscilloscope patterns.

For the particular receiver tested, the over-all time constant, corresponding to that which occurs in the analysis, was measured and found to be 80 microseconds.

In calculating the envelope of the interference pattern so that it may be compared with the experimental pattern as seen on the screen of the oscilloscope, the attenuation effect of the output system with respect to frequency is just as important as the attenuating effect of the deemphasis circuit in the receiver. Consequently, the envelope of the calculated interference pattern is proportional to the magnitude of Eq. (9) multiplied by the frequency response curve of the output system.

Since the interference frequency has a maximum value equal to the frequency deviation, the shape of the interference pattern envelope depends on the deviation. This envelope has been plotted for the two representative deviations of 75 and 15 kc and for the measured time constant of 80 microseconds in both cases. These plots are given in Fig. 3.

Photographs of the experimentally obtained patterns for the same two deviations and time constant are shown in Fig. 4. In these photographs, it is possible to see the actual variation in the interference frequency. This variation is sinusoidal and is represented by the changing density of the lines crossing the horizontal axis. It can be seen that the portion of the pattern having the greater density of lines is of a lower amplitude. This is consistent with the attenuating effects of the output system and the deemphasis circuit. To correlate the effects of the various attenuating factors on the interference pattern envelope, Fig. 5 is included.

Conclusions

The quantitative results are given in Fig. 6. Before the significance of the results is discussed, it might be well to elaborate on how the theoretical line, which indicates N/S as a function of a, is obtained. The theoretical line is obtained from Eq. (10) using the measured value of time constant equal to 80 microseconds. The value of interference frequency used was obtained graphically from the calculated interference pattern envelope for the 75-kc deviation. The interference frequency corresponding to the maximum point in the envelope was used in the calculations.

The measured and theoretical results deviate somewhat for increasing values of a. This is justified by the basic assumption that a is considered to be much less than unity.

The variations in measured results for different values of desired input signal are probably attributable to a slight error in the decade attenuation box on the f-m signal generator used. For small values of a, where the theoretical line is more accurate, these variations are of little importance.

In conclusion, the significant result of this investigation is that calculations relative to interference made on the basis of an idealized frequency-modulation receiver have been applicable to a representative commercial receiver.

This article is based on a bachelor's thesis submitted to the Department of Electrical Engineering of Massachusetts Institute of Technology. The author wishes to acknowledge the generous assistance of Professor L. B. Arguimbau, who supervised the thesis.

REFERENCES

(1) Reich, H. J., *Communications*, Aug. 1940.
(2) Pollack, Dale, M.I.T. E.E. Doctorate Thesis, 1940.
(3) Brainard, J. G., "Ultra-High Frequency Techniques," D. Van Nostrand, New York, 1942, p. 233.

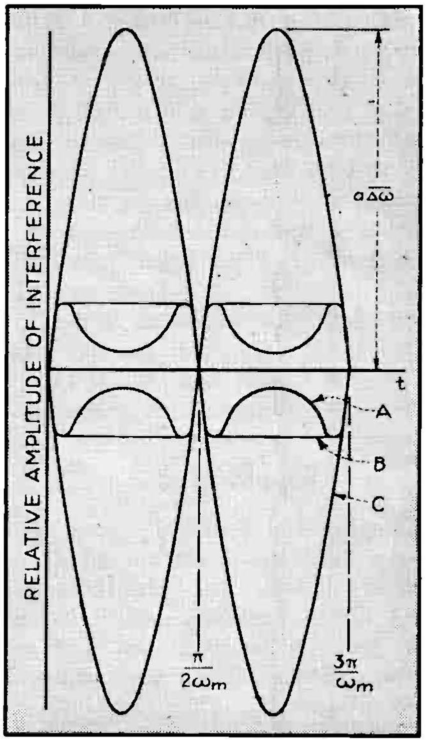

FIG. 5—Interference pattern envelopes for various attenuating factors. A—deemphasis circuit and output system: B—deemphasis circuit alone: C—theoretical envelope for no attenuating factors

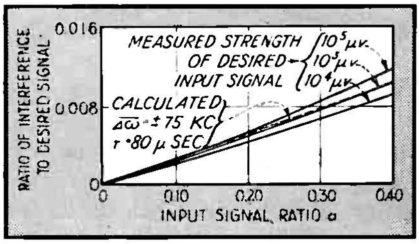

FIG. 6—Ratio of interfering signal to desired signal in the audio output of a representative f-m receiver, plotted as a function of the input signal ratio